Special IFIP TC7 Online Lectures; March 22nd 2023 , 18:00 (CET)

**Hadamard Semidifferentials,
Semidifferential of Parametrized Minima,
and Applications to
Shape and Topological Derivatives**

Michel C. Delfour

Centre de recherches mathématiques and Département de mathématiques et de statistique,
Université de Montréal, Montréal, Canada
michel.delfour@umontreal.ca

Abstract: Motivated by the notions of shape and topological derivatives, we revisit the Hadamard semidifferential, for which a complete semidifferential calculus is available, including the chain rule. Among its numerous applications, it coincides with the *conical derivative* of Mignot [Contrôle dans les inéquations variationelles elliptiques, J. Funct. Anal., 22 (1976), 130–185] and it is a natural tool for differentiation along trajectories in automatic differentiation. For real-valued functions we recall the *generalized directional derivative* (an upper semidifferential) for which some form of differential calculus is restored by going to subdifferentials. Both families of functions contain the convex functions, but they are not contained in one another. The choice is problem dependent, but the Hadamard semidifferential is more suitable for our purpose.

The object of this lecture is the differentiation of the infimum of parametrized objective functions with respect to the parameters as in Danskin [The theory of max-min, with applications, SIAM J. on Appl. Math. (4) **14** (1966), 641–644] who obtained a semidifferential equal to the infimum over the set of minimizers of the one-sided directional derivative with respect to the parameters.

Yet, in applications to the topological and shape derivatives of the compliance, examples reveal the possible occurrence of an extra negative term: the so-called *polarization term* in Mechanics.

The object of this lecture is to introduce new theorems that can accommodate the occurrence of an extra term. For the shape derivative, the associated technique is a change of variable to work on the fixed initial domain; for the topological derivative, it is an extension over the hole created by the topological perturbation of the domain.

This work has applications to compliance problems and to eigenvalue problems where the first eigenvalue is not simple.(see Delfour, [One-sided Derivative of Parametrized Minima for Shape and Topological Derivatives, SIAM J. Control Optim., accepted December 2022]).

Similar considerations apply to *constrained objective functions* via the one-sided derivative of the *minimax of a parametrized Lagrangian* (see Delfour [Topological Derivative of State Constrained Objective Functions: a Direct Approach, SIAM J. on Control and Optim. (1) **60** (2022), 22–47]).